Imaginary and Complex Numbers

Imaginary numbers are the square roots of negative numbers. These numbers can all be written in the form bi where b is a nonzero real number and i, called the **imaginary unit**, represents $\sqrt{-1}$. Some examples of imaginary numbers are the following:

- $2i$
- $-5i$
- $-\frac{i}{3}$ or $-\frac{1}{3}i$
- $i\sqrt{2}$ (Write the i in front of the radical symbol for clarity.)
- $\frac{i\sqrt{3}}{2}$ or $\frac{\sqrt{3}}{2}i$

$i = \sqrt{-1}$

$i^2 = -1$

Given that $i = \sqrt{-1}$, you can conclude that $i^2 = -1$. This means that the square of any imaginary number is a negative real number. When squaring an imaginary number, use the power of a product property of exponents: $(ab)^m = a^m \cdot b^m$.

$-\sqrt{-81}$

$-9i$

$\sqrt{-24}$

$\sqrt{-4}\cdot\sqrt{6}$

$2i\sqrt{6}$

$\sqrt{-18}$

$\sqrt{-9}\cdot\sqrt{2}$

$3i\sqrt{2}$

$5x^2 + 1 = -124$

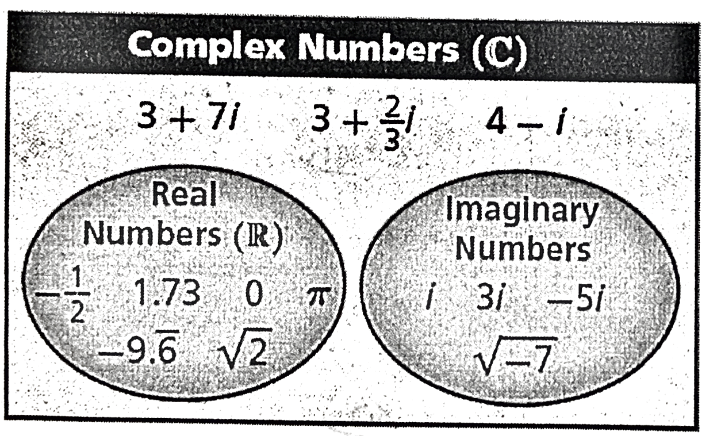
$\frac{-125}{5}$

$x^2 = -25$

$x = \pm 5i$

Objective: Perform operations with complex numbers

A <u>complex number</u> is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers C.



Every complex number has a <u>real part</u> a and an <u>imaginary part</u> b.

Real part		Imaginary part
↓		↓
a	**+**	***bi***

Two complex numbers are equal **if and only if** their real parts are ___equal___ and their imaginary parts are ___equal___.

Adding and subtracting complex numbers is similar to adding and subtracting variable expressions with like terms. **Simply combine the real parts, and combine the imaginary parts.**

You can multiply complex numbers by using the Distributive Property and treating the imaginary parts as like terms. **Simplify by using the fact $i^2 = -1$.**

Adding and Subtracting:

Match the problem on the left with the simplified expression it is equal to on the right.

1. $(4+2i)+(-6-7i)$ C.
   $-2-5i$
2. $(5-2i)-(-2-3i)$ B
   $5-2i+2+3i$ $7+i$
3. $(1-3i)+(-1+3i)$ E.
4. $(-3+5i)+(-6i)$ A
   $-3-i$
5. $2i-(3+5i)$ D.

A. $-3-i$

B. $7+i$

C. $-2-5i$

D. $-3-3i$

E. 0

Multiplying

Example 1: $-2i(2-4i)$

$-4i+8i^2$

$-8-4i$

Example 2: $(3+6i)(4-i)$

$12-3i+24i-6i^2$

$18+21i$

Example 3: $(2+9i)(2-9i)$

$4-81i^2$

85

Example 4: $(-5i)(6i)$

$-30i^2$

30

Match each product on the right with the corresponding expression on the left.

A. $(3-5i)(3+5i)$ $9-25i^2$ B $-16+30i$

B. $(3+5i)(3+5i)$ $9+30i+25i^2$ D -34

C. $(-3-5i)(3+5i)$ $-9-25i^2$ $-30i$ A 34

D. $(3-5i)(-3-5i)$ C. $16-30i$

-9 ~~i~~ $+25i^2$

Power	Expansion	Result
i	×	i
i^2	×	-1
i^3	$i^2 \cdot i \to (-1)i$	$-i$
i^4	$i^2 \cdot i^2 \to (-1)(-1)$	1
i^5	$i^4 \cdot i \to 1 \cdot i$ $i^2 \cdot i^2 \cdot i \to (-1)(-1)i$	i
i^6	$i^4 \cdot i^2 \to (1)(-1)$ $i^2 \cdot i^2 \cdot i^2 \to (-1)^3$	-1
i^7	$(i^2)^3 i \to (-1)^3 i$	$-i$
i^8	$(i^4)^2 \to (1)^2$	1

Simplify the following based on the table you created:

i^9
$(i^2)^4 i$
$(-1)^4 i$
$1 \cdot i$
i

i^{14}
$(i^2)^7$
$(-1)^7$
-1

i^{20}
$(i^2)^{10}$
$(-1)^{10}$
1

i^{25}
$(i^2)^{12} i$
$(-1)^{12} i$
i

$6i^{63}$
$6 \cdot (i^2)^{31} i$
$-6i$

$-2i^{17} + 5i^3$
$-2(i^2)^8 i + 5(i^2)i$
$-2i - 5i$
$-7i$

Homework: pg. 542 (7- 18)