16.1 Dilations

Essential Question: How does a dilation transform a figure?

⊘ Explore 1 Investigating Properties of Dilations

A **dilation** is a transformation that can change the size of a polygon but leaves the shape unchanged. A dilation has a *center of dilation* and a *scale factor* which together determine the position and size of the image of a figure after the dilation.

Use $\triangle ABC$ and its image $\triangle A'B'C'$ after a dilation to answer the following questions.
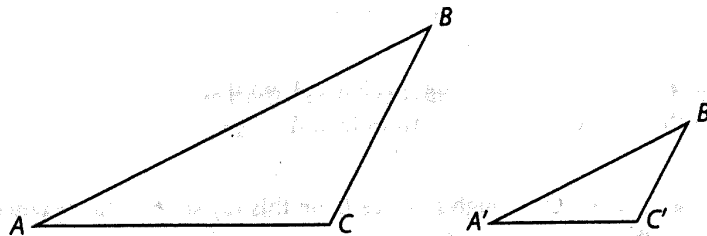
Ⓐ Use a ruler to measure the following lengths. Measure to the nearest tenth of a centimeter.

$AB = \boxed{6}$ cm $\quad A'B' = \boxed{3}$ cm

$AC = \boxed{4}$ cm $\quad A'C' = \boxed{2}$ cm

$BC = \boxed{3}$ cm $\quad B'C' = \boxed{1.5}$ cm

Ⓑ Use a protractor to measure the corresponding angles.

$m\angle A = \boxed{25°} \quad m\angle A' = \boxed{25°}$

$m\angle B = \boxed{35°} \quad m\angle B' = \boxed{35°}$

$m\angle C = \boxed{120°} \quad m\angle C' = \boxed{120°}$

Ⓒ Complete the following ratios

$\dfrac{A'B'}{AB} = \dfrac{\boxed{3}}{\boxed{6}} = \boxed{\frac{1}{2}} \qquad \dfrac{A'C'}{AC} = \dfrac{\boxed{2}}{\boxed{4}} = \boxed{\frac{1}{2}} \qquad \dfrac{B'C'}{BC} = \dfrac{\boxed{1.5}}{\boxed{3}} = \boxed{\frac{1}{2}}$

$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} \qquad \frac{length \ of \ image}{length \ of \ pre\text{-}image}$$

Reflect

1. What do you notice about the corresponding sides of the figures? What do you notice about the corresponding angles?

   _they are proportional_

   they are the same

2. **Discussion** What similarities are there between reflections, translations, rotations, and dilations? What is the difference?

   Sim: Ongles stay the same. all are transformations

   diff: don't change size

   → _change size_

⊘ Explore 2 Dilating a Line Segment

The dilation of a line segment (the pre-image) is a line segment whose length is the product of the scale factor and the length of the pre-image.

Use the following steps to apply a dilation by a factor of 3, with center at the point O, to $\overleftrightarrow{AC}$.
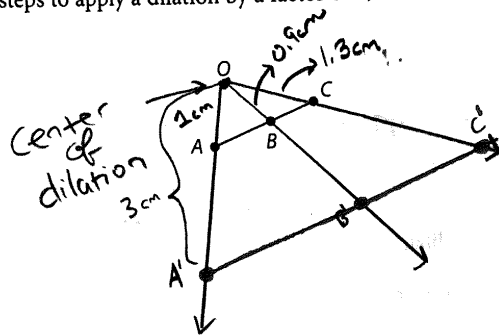
(A) To locate the point A', draw a ray from O through A. Place A' on this ray so that the distance from O to A' is three times the distance from O to A.

(B) To locate point B', draw a ray from O through B. Place B' on this ray so that the distance from O to B' is three times the distance from O to B.

(C) To locate point C', draw a ray from O through C. Place C' on this ray so that the distance from O to C' is three times the distance from O to C.

(D) Draw a line through A', B', and C'.

(E) Measure $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Measure $\overline{A'B'}$, $\overline{A'C'}$, and $\overline{B'C'}$. Make a conjecture about the lengths of segments that have been dilated.
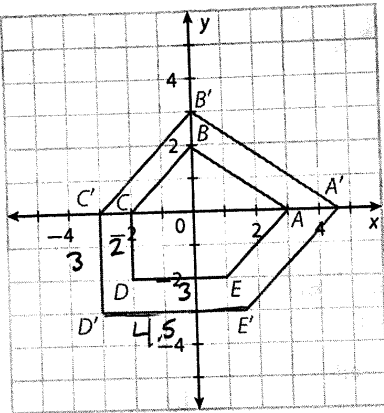
_____

Reflect

3. Make a conjecture about the length of the image of a 4 cm segment after a dilation with scale factor k. Can the image ever be shorter than the preimage?

_____

_____

4. What can you say about the image of a segment under a dilation? Does your answer depend upon the location of the segment? Explain
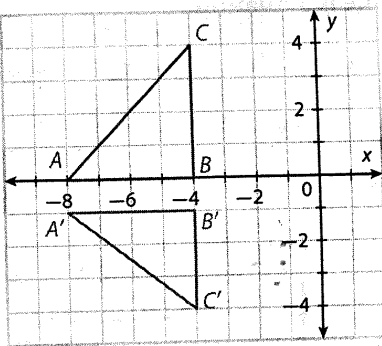
_____

_____

Determine if the transformations are dilations.

5.

yes
scale factor: $\frac{3}{2}$

6.

not a dilation

⚷ Explain 2 Determining the Center and Scale of a Dilation

When you have a figure and its image after dilation, you can find the center of dilation by drawing lines that connect corresponding vertices. These lines will intersect at the center of dilation.

Example 2 Determine the center of dilation and the scale factor of the dilation of the triangles.

(A) Draw $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$, and $\overleftrightarrow{CC'}$. The point where the lines cross is the center of dilation. Label the intersection O. Measure to find the scale factor.
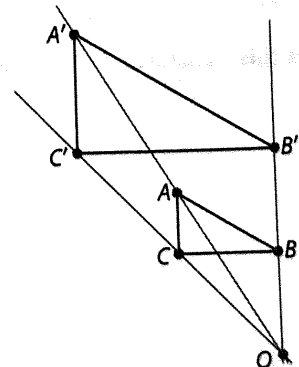

$OA = 25$ mm $OB = 13$ mm $OC = 19$ mm

$OA' = 50$ mm $OB' = 26$ mm $OC' = 38$ mm

The scale factor is 2 to 1.

Ⓑ Draw $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$, and $\overleftrightarrow{CC'}$. Measure from each point to the intersection O to the nearest millimeter.

$OA = $ 6

$OA' = $ ≈ 2.5 → 3

$OB = $ ≈ 4

$OB' = $ 2

$OC = $ 5.5

$OC' = $ 2.75

$\dfrac{\text{length of image}}{\text{length of preimge}}$

The scale factor is $\dfrac{1}{2}$.
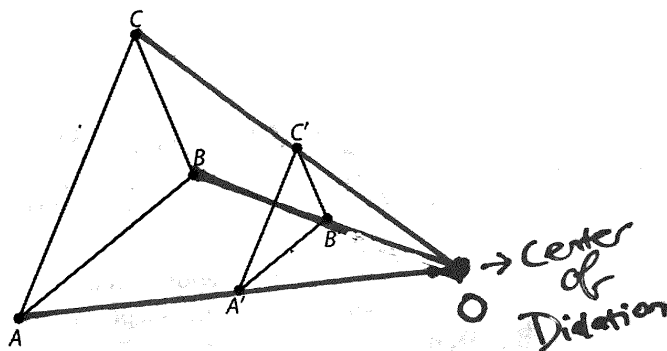
→ center of Dilation

Reflect

7. For the dilation in Your Turn 5, what is the center of dilation? Explain how you can tell without drawing lines.

Your Turn

8. Determine the center of dilation and the scale factor of the dilation.

$OA' = $ 1 inch $\text{cm, } OA = $ 3 inches

The scale factor of the dilation is $\dfrac{1}{3}$.

Center of dilation

scale factor < 1 image is smaller than the preimage
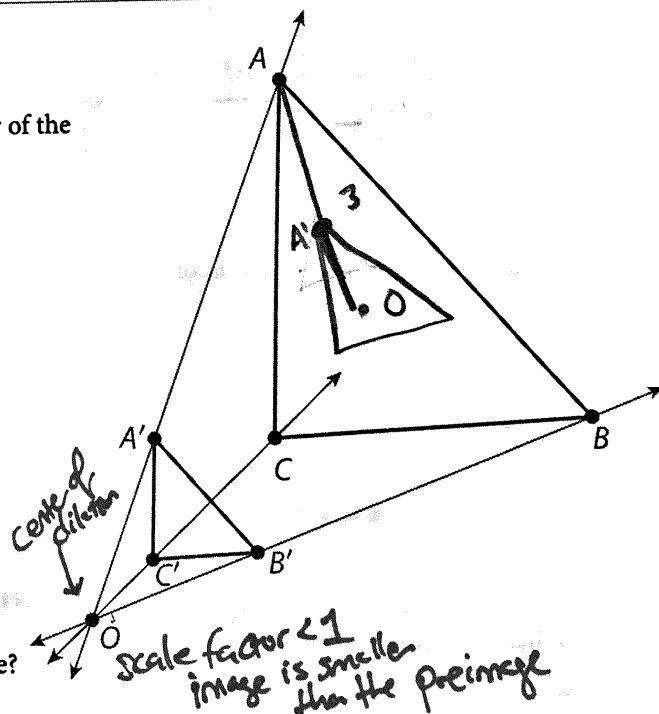
💬 Elaborate

9. How is the length of the image of a line segment under a dilation related to the length of its preimage?

10. **Discussion** What is the result of dilating a figure using a scale factor of 1? For this dilation, does the center of dilation affect the position of the image relative to the preimage? Explain.

stays the same   Changes position not the size

11. Essential Question Check-In In general how does a dilation transform a figure?
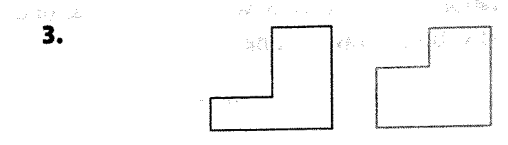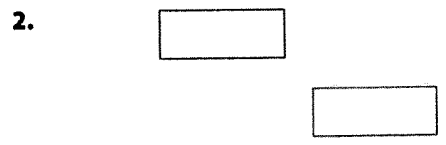

☆ Evaluate: Homework and Practice

1. Consider the definition of a dilation. A dilation is a transformation that can change the size of a polygon but leaves the shape unchanged. In a dilation, how are the ratios of the measures of the corresponding sides related?

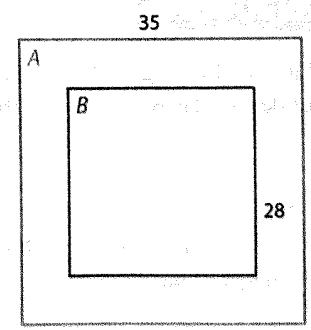**Tell whether one figure appears to be a dilation of the other figure Explain.**

2.

3.

4. Is the scale factor of the dilation of $\triangle ABC$ equal to $\frac{1}{2}$? Explain.
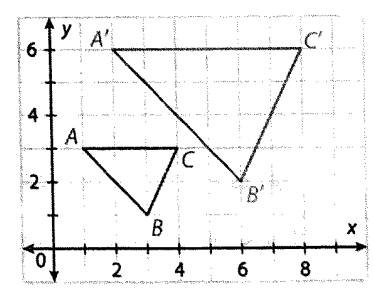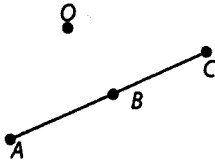
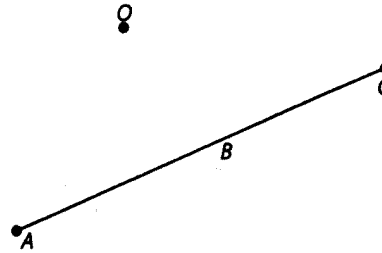5. Square A is a dilation of square B.
   What is the scale factor?

   a. $\frac{1}{7}$

   b. $\frac{4}{5}$

   c. $\frac{5}{4}$

   d. 7

   e. $\frac{25}{16}$

6. Apply a dilation to $\overline{AC}$ with a scale factor of 2 and center at the point O.
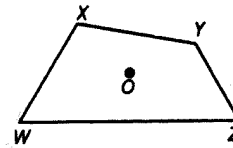
Q

C

B

A

7. Apply a dilation to $\overline{AC}$ with a scale factor of $\frac{1}{3}$ and center at the point O.
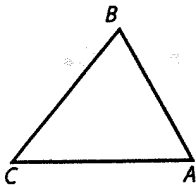
Q

C

B

A

8. What happens when a triangle is dilated using one of the vertices as the center of dilation?

9. Draw an image of $WXYZ$. The center of the dilation is O, and the scale factor is 2.
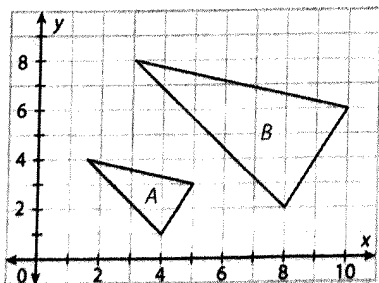
X

Y

O

W

Z

10. Draw an image of $\triangle ABC$. The center of dilation is C, and the scale factor is 1.5.

11. Compare dilations to rigid motions. How are they the same? How are they different?

B

C

A

Determine if the transformation of figure A to figure B on the coordinate plane is a dilation. Verify ratios of corresponding side lengths for a dilation.

12.

y

8

6

4

2

B

A

x

0   2   4   6   8   10

13.

y

6

4

2

A

B

0   2   4   6

x