## Imaginary and Complex Numbers, continued

Key

Multiply: $(8-3i)^2 + (4-i)$

$(8-3i)(8-3i) + 4 - i$

$64 - 48i + 9i^2 + 4 - i$

$59 - 49i$

$-6(4-6i) + (3i)^3$

$-24 + 36i + 27i^3$

$-24 + 36i + -27i$

$-24 + 9i$

Simplify: $3 + i^5$

$i^5$

$(i^2)^2 i$

$i$

$3 + i$

$2 + i^{206}$

$(i^2)^{103}$

$(-1)^{103}$

$2 - 1$

$1$

$3i^{204}$

$3(i^2)^{102}$

$3$

$5i^{35} - 6i^{14} + i^7$

$5(i^2)^{17} i - 6(i^2)^7 + (i^2)^3 i$

$-5i + 6 - i$

$6 - 6i$

### Division with Complex Numbers

- To be considered "simplified", fractions should not have square roots in the denominator.
- To get rid of square roots in the denominator we need to "rationalize the denominator" or multiply both top and bottom by a root.

Simplify: $\frac{\sqrt{3}}{\sqrt{3}}$ $1$ $\quad \frac{\sqrt{5}}{\sqrt{5}}$ $1$ $\quad \frac{\sqrt{2}}{\sqrt{2}}$ $1$ $\quad \frac{\sqrt{6}}{\sqrt{6}}$ $1$

Multiply by a Special form of one to rationalize: $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$ $\quad \frac{2}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{2\sqrt{2}}{2}$ $\sqrt{2}$

$\frac{5}{\sqrt{3}}$ $\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{5\sqrt{3}}{3}$

$\frac{7}{\sqrt{7}}$ $\frac{\sqrt{7}}{\sqrt{7}}$ $\frac{7\sqrt{7}}{7}$ $\sqrt{7}$

$\frac{8}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{8\sqrt{2}}{2}$ $4\sqrt{2}$

$\frac{10}{\sqrt{6}}$ $\frac{\sqrt{6}}{\sqrt{6}}$ $\frac{10\sqrt{6}}{6}$ $\frac{5\sqrt{6}}{3}$

What about something like $\frac{1}{5+\sqrt{2}}$? $\frac{5-\sqrt{2}}{5-\sqrt{2}}$ $\frac{5-\sqrt{2}}{25-2}$

$\frac{5-\sqrt{2}}{23}$

$\frac{3}{6-\sqrt{3}}$ $\frac{6+\sqrt{3}}{6+\sqrt{3}}$ $\frac{18+3\sqrt{3}}{36-3}$

$\frac{18+3\sqrt{3}}{33}$ $\frac{6+\sqrt{3}}{11}$

Because the imaginary unit represents a square root, you must rationalize any denominator that contains an imaginary unit.

$\frac{3+10i}{5i} \cdot \frac{i}{i}$ $\frac{3i+10i^2}{5i^2} = \frac{-10+3i}{-5}$

$2 - \frac{3}{5}i$

$\frac{3+8i}{-i} \cdot \frac{i}{i}$ $-8+3i$

$\frac{2-4i}{2i} \cdot \frac{i}{i}$ $-2-i$

A *complex conjugate pair* has real parts that are equal and imaginary parts are opposites.
The **complex conjugate** of any complex number $a + bi$ is the complex number $a - bi$.

*Example of a complex conjugate pair:*

*Write your answer in the form a + bi*

$\frac{2+8i}{4-2i} \cdot \frac{4+2i}{4+2i}$ $\quad \frac{8+4i+32i+16i^2}{16-4i^2}$

$\frac{-8+36i}{20}$ $\quad \frac{-8}{20}+\frac{36}{20}i$

$-\frac{2}{5}+\frac{9}{5}i$

$\frac{3-i}{2-i} \quad \frac{2+i}{2+i}$ $\quad \frac{6+3i-2i-i^2}{4-i^2}$ $\quad \frac{7+i}{5}$ $\quad \frac{7}{5}+\frac{1}{5}i$

$\frac{1+3i}{2-5i} \quad \frac{2+5i}{2+5i}$ $\quad \frac{2+5i+6i+15i^2}{4-25i^2} = \frac{-13+11i}{29}$

$\frac{-13}{29}+\frac{11}{29}i$

$\frac{4+2i}{3-i} \cdot \frac{3+i}{3+i}$ $\quad \frac{12+10i+2i^2}{9-i^2} = \frac{10+10i}{10}$ $\quad 1+i$

*Simplify the numerator first:*

$\frac{(1+2i)(1-2i)}{6-i}$ $\quad \frac{1-4i^2}{6-i}$ $\quad \frac{5}{6-i} \cdot \frac{6+i}{6+i}$ $\quad \frac{30+5i}{36-i^2}$

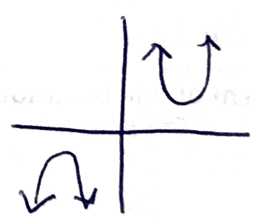
$\frac{30+5i}{37}$ $\quad \frac{30}{37}+\frac{5}{37}i$

$\frac{(8-i)(8+i)}{5-4i}$ $\quad \frac{64-i^2}{5-4i}$ $\quad \frac{65}{5-4i} \cdot \frac{5+4i}{5+4i}$ $\quad \frac{325+260i}{25-16i^2}$

$\frac{325+260i}{41}$

$\frac{325}{41}+\frac{260}{41}i$

What would a graph that has imaginary zeros/roots look like?



Find the zeros/roots of each function using the quadratic formula:

$y = x^2 + 10x + 26$

$0 = x^2+10x+26$

$\frac{-10 \pm \sqrt{100-4(1)(26)}}{2(1)}$

$\frac{-10 \pm \sqrt{-4}}{2}$

$-5 \pm i$

$y = x^2 + 4x + 12$

$\frac{-4 \pm \sqrt{16-4(1)(12)}}{2(1)}$

$\frac{-4 \pm \sqrt{-32}}{2}$

$\frac{-4 \pm 4i\sqrt{2}}{2}$

$-2 \pm 2i\sqrt{2}$

$y = 2x^2 + 6x + 10$

$0 = 2x^2+6x+10$

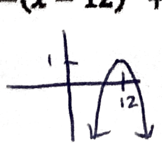
$0 = x^2+3x+5$

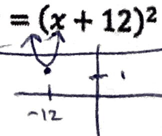
$\frac{-3 \pm \sqrt{9-4(1)(5)}}{2(1)}$

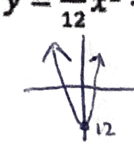
$\frac{-3 \pm i\sqrt{11}}{2}$

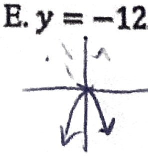
Circle the equations of the graphs with imaginary zeros. Think about transformations of functions.

A. $y = 4x^2 + 12$ B. $y = -(x-12)^2 + 1$ C. $y = (x+12)^2 + 1$ D. $y = \frac{1}{12}x^2 - 12$ E. $y = -12x^2$







# *Operations with Complex Numbers*

Exar

**Add or subtract. Write the result in the form $a + bi$.**

1. $(8 - i) - (-5 - 4i)$

2. $(2 - 11i) - (10 + 6i)$

3. $\left(\frac{1}{2} + \frac{3}{4}i\right) + \left(-\frac{1}{4} - \frac{5}{4}i\right)$

4. $(-6 - i) + (1 + 3i)$

5. $(-2 - 2i) + (8 - 6i)$

Multiply or divide. Write the result in the form $a + bi$.

6. $\dfrac{-3 + 7i}{1 + 8i}$

7. $(-4 - 9i)(8 + 2i)$

8. $\dfrac{5 + i}{2 - i}$

**Simplify.**

9. $i^{24} - i^{13} + i^{12}$

10. $-4i^{13}$

11. $6 - 4i^{18}$

**Solve.**

12. In a circuit, the voltage, $V$, is given by the formula $V = IZ$, where $I$ is the current and $Z$ is the impedance. Both the current and impedance are represented by complex numbers. Find the voltage if the current is $3 + 2i$ and the impedance is $4 - i$.