Semester Exam Review Packet Key

*This packet is not necessarily comprehensive. In other words, this packet is not a promise in terms of level of difficulty or full scope of material.

Polynomials

1. Classify by degree and number of terms: $7x^2 - 5x^3y^3$

binomial degree 6

2. Which is the correct classification of $25xyz + 6x^2+1$?

A. binomial with a degree of 2
B. binomial with a degree of 3
C. trinomial with a degree of 2
D. trinomial with a degree of 3

3. Add. $(3x^4 - 9x^3 + 5x^2 - x + 7) + (3 + 4x^4 + 3x - x^3 + 3x^2)$

$7x^4 - 10x^3 + 8x^2 + 2x + 10$

4. Subtract. $(7x^3 - 4x - x^2 + 2 - 20x^5 - 6x^4) - (8 + 3x^3 - 2x - 12x^5 + 7x^2 - 8x^4)$

$-8x^5 + 2x^4 + 4x^3 - 8x^2 - 2x - 6$

5. Simplify: $3(x - 1)^3$

$3(x-1)(x-1)(x-1)$
$3(x^2-2x+1)(x-1)$
$3(x^3-3x^2+3x-1)$
$3x^3-9x^2+9x-3$

6. Simplify: $(x^5 - 2)(x^5 + 5)$

$x^{10} + 5x^5 - 2x^5 - 10$
$x^{10} + 3x^5 - 10$

7. Simplify: $(x + 5)(x + 7)$

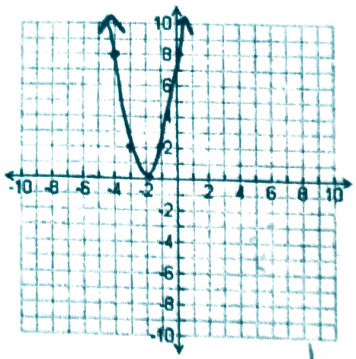
$x^2+7x+5x+35$
$x^2+12x+35$

8. Simplify: $(x^3y + 5)^2$

$(x^3y+5)(x^3y+5)$
$x^6y^2 + 5x^3y + 5x^3y + 25$
$x^6y^2 + 10x^3y + 25$

Transformations of Functions

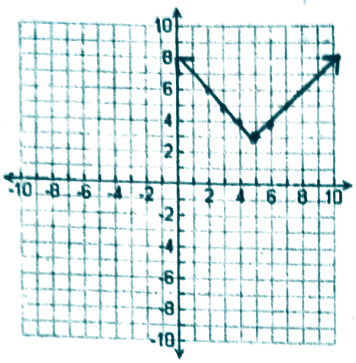
Graph the following transformations. Describe each transformation in words.

9. 

$f(x) = x^2$

$2f(x+2)$

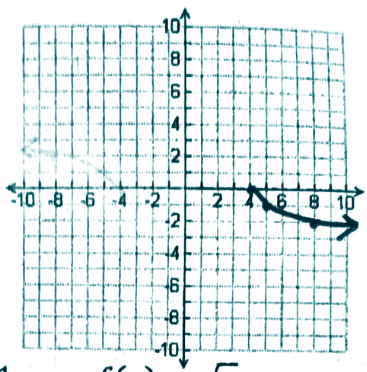
left 2, vertical stretch by 2

10. 

$f(x) = |x|$

$f(x-5)+3$

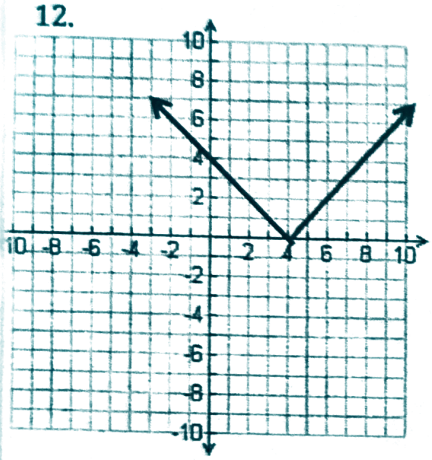
right 5 up 3

11. 

$f(x) = \sqrt{x}$

$-f(x-4)$

right 4, reflect across x axis

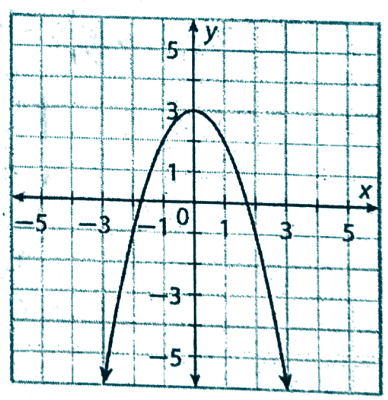
12. 

Increasing Interval(s): $(4, \infty)$

Decreasing Interval(s): $(-\infty, 4)$

$f(4) =$ 0

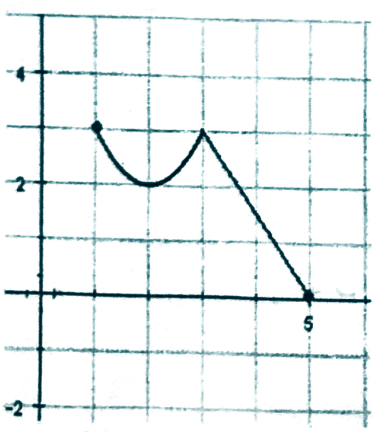
$f(?) = 3$ 1, 7

13. 

Domain (interval notation): $(-\infty, \infty)$

Range (interval notation): $(-\infty, 3]$

Maximum (only the y value!): 3

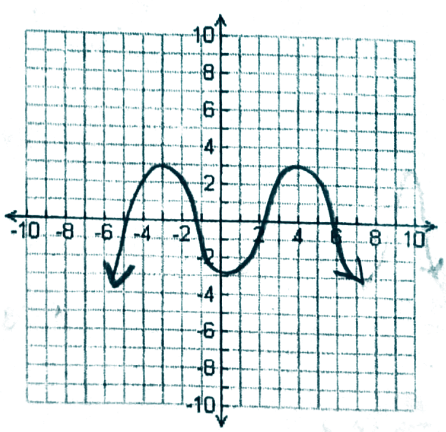
14. 

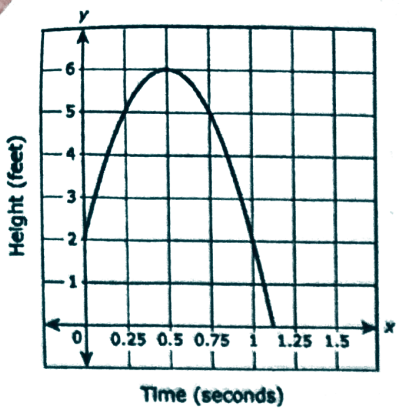
Domain (interval notation): $[1, 5]$

Range (interval notation): $[0, 3]$

Minimum (only the y value!): 0

15. Draw a graph that is increasing then decreasing then increasing then decreasing and has a domain of all real numbers and a range of $(-\infty, 3]$





Use the graph to estimate the average rate of change of the height of the ball for the first 0.25 seconds after being hit.

A. 0.75 feet per second
B. 3.0 feet per second
C. 12 feet per scond
D. 20 feet per second.

(0,2) (.25,5) $\frac{5-2}{.25-0} = \frac{3}{.25} = 12$

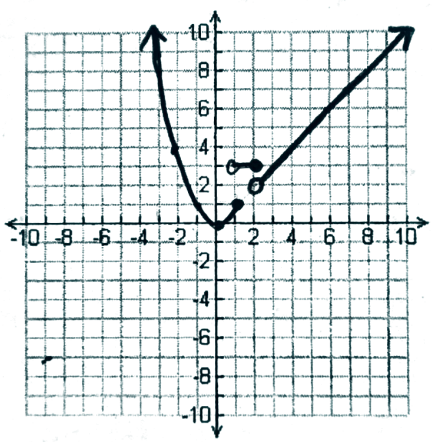
17. For the following piecewise function:

$$f(x) = \begin{cases} -\frac{1}{3}x + 1, & x \le 0 \\ (x-5)^2, & x > 0 \end{cases}$$

Evaluate: f(3)= $(3-5)^2$ = 4 f(0)= $-\frac{1}{3}(0)+1$ = 1 f(-6)= $-\frac{1}{3}(-6)+1$ = 3 f(8)= $(8-5)^2$ = 9

18. Graph the piecewise function

$$f(x) = \begin{cases} x^2, & x \le 1 \\ 3, & 1 < x \le 2 \\ x, & x > 2 \end{cases}$$



Exponents

19. $\frac{60b^6b^3a^8e^2}{4a^8e^{-2}}$ → $15b^3e^4$

20. $\left(\frac{2c^{-3}a^4}{a^{10}}\right)^{-2}$ → $\left(\frac{2c^{-3}a^{10}}{a^4}\right)^2$ → $\left(\frac{a^6c^3}{2}\right)^2$ → $\frac{a^{12}c^6}{4}$

21. $-6^2 \cdot 2^{-2} \cdot 8^{\frac{2}{3}}$ → $(\sqrt[3]{8})^2$

$-36 \cdot \frac{1}{4} \cdot 4$

-36

22. $\sqrt[3]{(27y^3)^4}$ → $(27y^3)^{\frac{4}{3}}$ → $\left[(27y^3)^{\frac{1}{3}}\right]^4$ → $(3y)^4$ → $81y^4$

23. $\left(x^{\frac{1}{2}}\right)^4 \sqrt{x^6} \frac{\left(x^{\frac{1}{4}}\right)^8}{\sqrt[3]{x^3}}$

$x^2 \, x^3 \, \frac{x^2}{x}$

$\frac{x^7}{x} = x^6$

24. $25^{\frac{5}{2}} + 32^{\frac{1}{5}} - (20)^0$

$5^5 + 2 - 1$

3126

25. $4^{2x+3} = 256$

$4^{2x+3} = 4^4$

$2x+3=4$

$2x=1$

$x=1/2$

26. $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$-2x+1+(-2x)-3=-x$

$-4x-2=-x$

$-2=3x$

$-2/3=x$

Exponentials

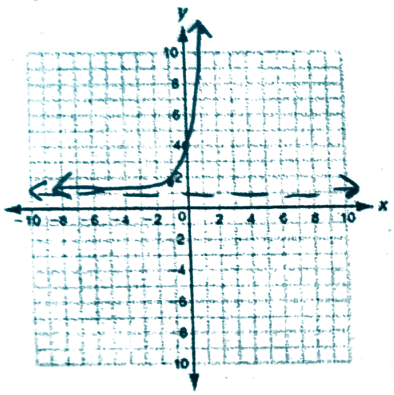
27. What values of b in the form $y = ab^x$ will give an equation for exponential growth? What about exponential decay?

If $b>1$, then it is exponential growth

If $b<1$, then it is exponential decay

28. Graph the Exponential Function and describe its key features.

$y = 3(4)^x + 1$



Growth or Decay?	Growth
Domain	$(-\infty, \infty)$
Range	$(1, \infty)$
y – intercept	4
Asymptote	$y=1$
End Behavior	$x \to \infty, f(x) \to \infty$ $x \to -\infty, f(x) \to 1$

$y = (4)^x$

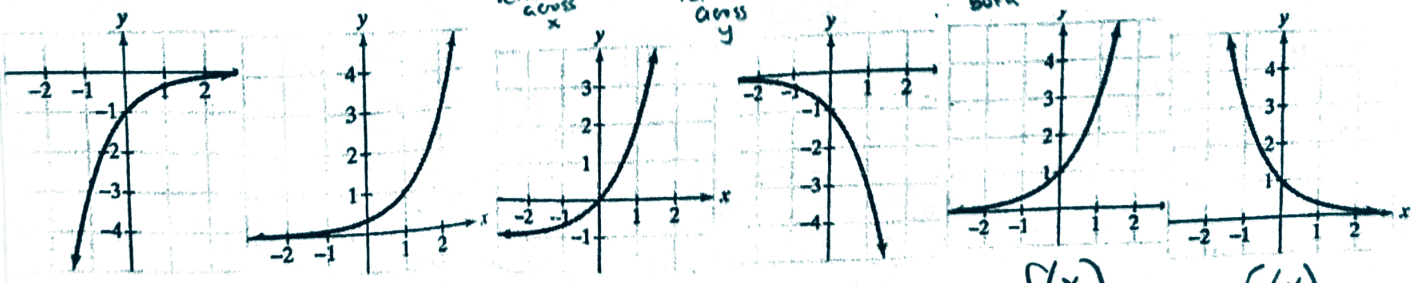
-1	$\frac{1}{4}$ $\frac{3}{4}$ $1\frac{3}{4}$
0	1 2 3 4
1	4 12 13

29. Match each function to a graph.

~~$f(x) = 3^x, g(x) = 3^{x-1}, h(x) = 3^x - 1,$~~

$F(x) = -3^x, G(x) = 3^{-x}, H(x) = -3^{-x}.$

reflect across x — reflect across y — reflect across both



H(x) g(x) h(x). F(x) f(x) G(x)

30. The value of a car can be modeled by the function $g(t) = 22500(0.554)^t$, where **t** is the number of years. Describe what is happening with the value of the car, using both numbers from the function in your explanation.

The car is originally worth $22,500 and is decreasing in value by 44.6% per year

31. Suppose 6,700,000 people watch the first episode of "Keeping Up with the Kardashians", but the number of viewers decreases by 3.5% each week.

a. Write an exponential function to model the situation.

$y = 6,700,000(.965)^x$ x = # of weeks

b. If the pattern continues, how many will watch the season finale, which is ten weeks later?

≈ 4,691,891 people

32. Jane's credit card company charges 20% interest per year, compounded quarterly. If Jane's credit card bill was originally $775, how much will the bill be after 4 years if she doesn't pay it off? Round your answer to the ~~nearest cent.~~

$y = 775\left(1 + \frac{.20}{4}\right)^{4t}$ t = # of years

$1691.73

33. Three scientists describe the amount of a radioactive substance, Q in grams, left after t years:

A: $500\left(\frac{1}{2}\right)^{\frac{t}{8}}$ B: $500(0.917)^t$ C: $385.548(0.917)^{t-3}$

a. Which expression highlights the half-life (the time taken for the radioactivity of a specified isotope to fall to half its original value) of the radioactive substance? What is the half life?

(A) 8 years

b. Which of the expressions highlights what the amount of radioactivity is after 3 years? How much is it after 3 years?

(C) 385.548

c. Which expression highlights the decay rate each year? What is the percent decay each year?

(B) 8.3%

34. Circle two below that are equivalent. Below your choice show by hand how you can convert one to the other.

(a.) $4(2)^{2t}$ (b.) 4^{t+1} c. $16(2)^t$ d. 8^{2t} (e.) $(2)^{2t+2}$

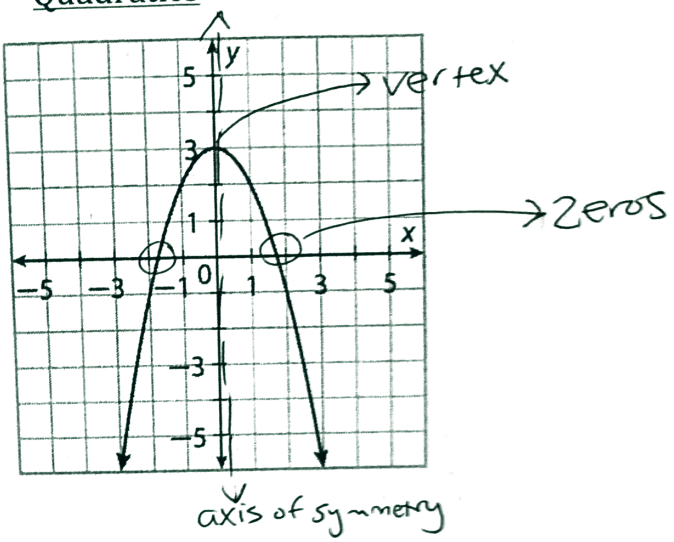
$4(2^2)^t$
$4 \cdot 4^t$
4^{t+1}

$4^t \cdot 4$

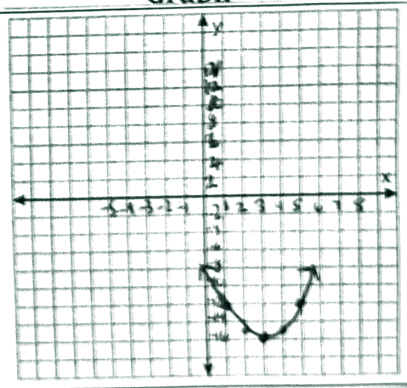
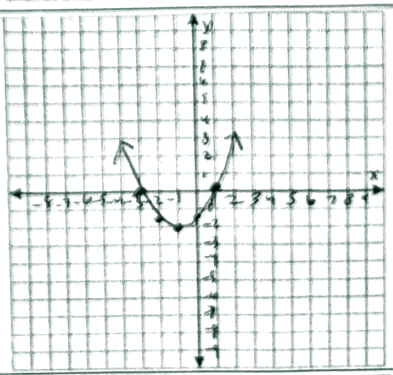
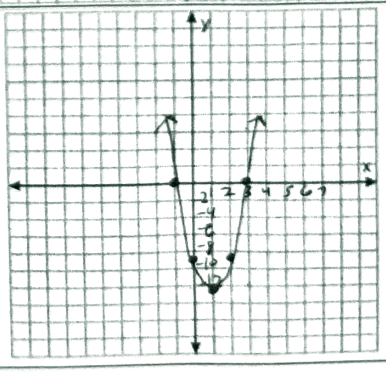
$2^{2t} \cdot 2^2$
$4^t \cdot 4$

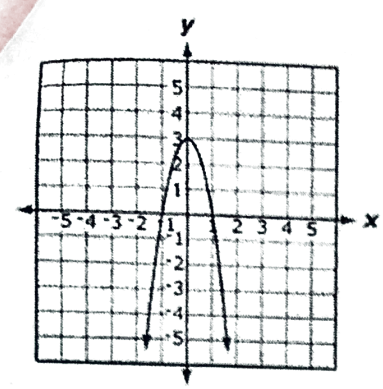
Quadratics

35. Label the vertex, zeros, and axis of symmetry on the graph.



36. Fill in the table.

Vertex Form	Intercept Form	Standard Form	Graph
$y = x^2 - 6x + \boxed{9} - 7 \boxed{-9}$ $y = (x-3)^2 - 16$	~~$y = a(x-r_1)(x-r_2)$~~ $y = (x-7)(x+1)$	mistake $y = 4x^2 - 6x - 7$	
$y = \frac{1}{2}(x+1)^2 - 2$	$y = \frac{1}{2}(x^2+2x-3)$ $y = \frac{1}{2}(x-1)(x+3)$	$y = \frac{1}{2}(x+1)(x+1) - 2$ $y = \frac{1}{2}(x^2+2x+1) - 2$ $y = \frac{1}{2}x^2 + x + \frac{1}{2} - 2$ $y = \frac{1}{2}x^2 + x - 1.5$	
$y = 3(x^2 - 2x + \boxed{1}) - 9 + \boxed{-3}$ $y = 3(x-1)^2 - 12$	$y = 3(x^2 - 2x - 3)$ $y = 3(x-3)(x+1)$	$y = 3x^2 - 6x - 9$	



37. Write the equation in intercept form, vertex form and standard form.

$y = -3(x+1)(x-1)$

$y = -3(x+0)^2 + 3$

$y = -3x^2 + 3$

38. Solve this equation using the methods listed below: $x^2 - 15x = -50$

Factoring

$x^2 - 15x + 50 = 0$

$(x-10)(x-5) = 0$

$x = 10$ or $x = 5$

Complete the Square

$x^2 - 15x + \boxed{56.25} = -50 + \boxed{56.25}$

$\sqrt{(x-7.5)^2} = \sqrt{6.25}$

$x - 7.5 = \pm 2.5$

$x = 7.5 \pm 2.5$

$x = 10$ or $x = 5$

Quadratic Formula

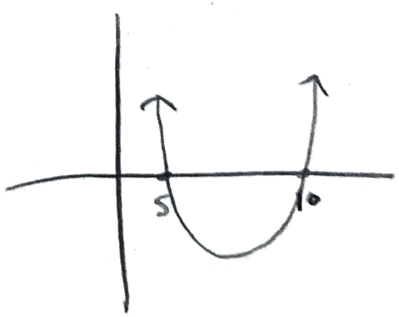
$x^2 - 15x + 50 = 0$

$$\frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(50)}}{2(1)}$$

$$\frac{15 \pm \sqrt{225 - 200}}{2} \rightarrow \frac{15 \pm \sqrt{25}}{2} \rightarrow \frac{15 \pm 5}{2}$$

$x = 10$ or $x = 5$

Graphing (SKETCH)



Solve using the method of your choice:

39. $3n^2 - 8n = -4$

$3n^2 - 8n + 4 = 0$

$(3n-2)(n-2) = 0$

$n = \frac{2}{3}$ or $n = 2$

40. $-9x^2 = 66x + 21$

$0 = 9x^2 + 66x + 21$

$0 = 3(3x^2 + 22x + 7)$

$0 = 3(3x+1)(x+7) = 0$

$x = -\frac{1}{3}$ $x = -7$

41. $4x^2 + 3x = 10$

$4x^2 + 3x - 10 = 0$

$4x^2 + 8x - 5x - 10 = 0$

$4x(x+2) - 5(x+2) = 0$

$(4x-5)(x+2) = 0$

$x = -2$ or $x = 5/4$

42. $5x^2 - 18x = 9$

$5x^2 - 18x - 9 = 0$

$$\frac{-(-18) \pm \sqrt{324 - 4(5)(-9)}}{10} \rightarrow \frac{18 \pm \sqrt{504}}{10} \rightarrow \frac{18 \pm 6\sqrt{14}}{10} \rightarrow \frac{9 \pm 3\sqrt{14}}{5}$$

43. What is the Discriminant and what does it tell us about how many solutions there are?

b^2-4ac (under the radical)

If $b^2-4ac > 0$ there are 2 real solutions
If $b^2-4ac = 0$ there is 1 real solution
If $b^2-4ac < 0$ there are no real solutions

44. The height in feet of a baseball seconds after being hit is given by $h(t) = -16t^2 + 64t + 3$.

a. What is the initial height of the baseball? 3 feet

b. What is the maximum height of the ball? When does this occur? (do not use a calculator)

$-\frac{b}{2a} \rightarrow \frac{-64}{2(-16)} = \frac{-64}{-32} = 2$ (2, 67)

67 feet after 2 seconds

$-16(2)^2+64(2)+3$
$-64+128+3$

45. The height of a flare fired from the deck of a ship in distress can be modeled by $h = -16t^2 + 104t + 56$, where h is the height in feet of the flare above water and t is the time in seconds.

Find the time it takes the flare to hit the water. Factor to solve. Do not use a calculator.

$0 = -16t^2+104t+56$
$0 = -8(2t^2-13t-7)$
$0 = -8(2t+1)(t-7)$

7 seconds

46. The length of a rectangle is 8 feet more than its width. The area of the rectangle is 84 square feet. Find its length and width. Draw a picture and set up an equation to solve.

$w(w+8) = 84$
$w^2+8w-84=0$
$(w+14)(w-6)=0$

$w = 6$

6 × 14

47. The height of an object moving in a parabolic path can be found by using the formula $h = -16t^2 + vt + s$, where v is the initial upwards velocity in feet per second and s is the starting height in feet. A basketball player shoots at the basket from a starting height of 6 feet and an upwards velocity of 20 feet per second.

Determine how long it takes for the basketball shot to drop through the basket that is mounted at a height of 10 feet. Factor to solve- do not use a calculator. (Hint: you will get two positive solutions- think about the graph to determine which one is the correct answer)

$h = -16t^2+20t+6$
$10 = -16t^2+20t+6$
$0 = -16t^2+20t-4$

$0 = -4(4t^2-5t+1)$
$0 = -4(4t-1)(t-1)$

$\frac{1}{4}$ second & 1 second

on way back down

48. $\frac{(1-2i)(1-2i)}{7i}$

$\frac{1-2i-2i+4i^2}{7i}$

$\frac{-3-4i}{7i} \cdot \frac{i}{i}$

$\frac{-3i-4i^2}{7i^2}$

$\frac{4-3i}{-7}$

$-\frac{4}{7}+\frac{3}{7}i$

49. $\frac{(6+i)(6-i)}{1+3i}$

$\frac{36-i^2}{1+3i} \rightarrow \frac{37}{1+3i} \cdot \frac{1-3i}{1-3i} = \frac{37-111i}{1-9i^2}$

$\frac{37-111i}{10}$

$\frac{37}{10}-\frac{111}{10}i$

$i^{600} + i^{602}$

$(i^2)^{300} + (i^2)^{301}$

$(-1)^{300} + (-1)^{301}$

$1 - 1 = 0$

51. $i^{24} - i^2$

$(i^2)^{12} - (-1)$

$1 + 1$

2

52. Solve: $5x^2 + x + 2 = 0$

$\frac{-1 \pm \sqrt{1-4(5)(2)}}{2(5)}$

$\frac{-1 \pm \sqrt{-39}}{10}$

$\frac{-1 \pm i\sqrt{39}}{10}$

$-\frac{1}{10} \pm \frac{\sqrt{39}}{10} i$

53. Solve: $9x^2 + 5 = -95$

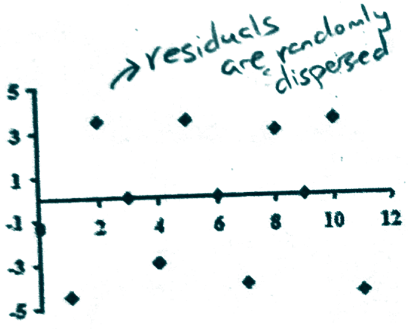
$\frac{9x^2 = -100}{9}$

$\sqrt{x^2} = \sqrt{\frac{-100}{9}}$

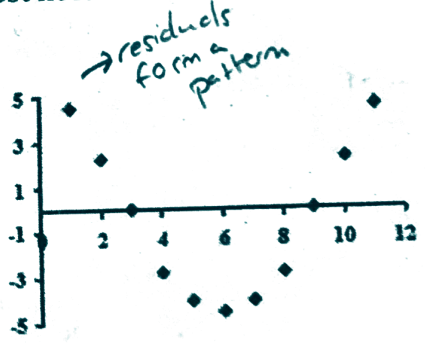
$x = \pm \frac{10}{3} i$

54. Which residual plot implies that a linear model is the best fit for the data?

a.



b.



55. What type of function is described by the table? Explain your answer.

Quadratic - second differences are the same

x	1	4	7	10	13
y	-4	11	44	95	164

+15 +33 +51 +69

+18 +18 +18

56. Which equation best fits the data?

A.) $y = -(x+2)^2 + 4$

B.) $y = (x-2)^2 + 4$

C.) $y = -(x-2)^2 + 4$

D.) $y = (x+2)^2 + 4$