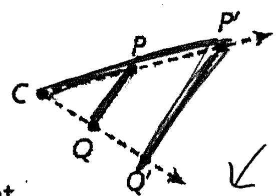
Dilations

Dilations

A dilation, or *similarity transformation,* is a transformation in which the lines connecting every point P with its image P' all intersect at a point C, called the center of dilation. $\frac{CP'}{CP}$ is the same for every point P.
The scale factor k of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. In the figure, $k = \frac{P'Q'}{PQ}$.



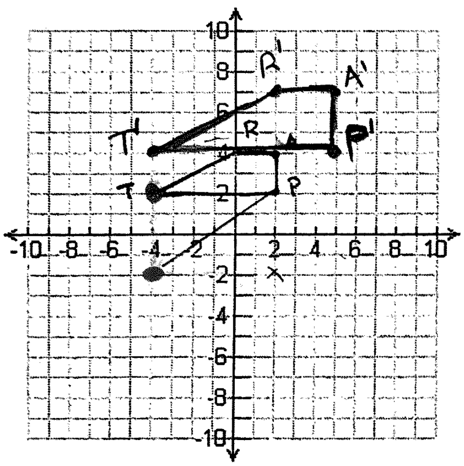
scale factor

$k = \frac{P'Q'}{PQ} = \frac{CP'}{CP}$

A dilation enlarges or reduces all dimensions proportionally. A dilation with a scale factor greater than 1 is an **enlargement**, or *expansion*. A dilation with a scale factor greater than 0 but less than 1 is a **reduction**, or *contraction*.

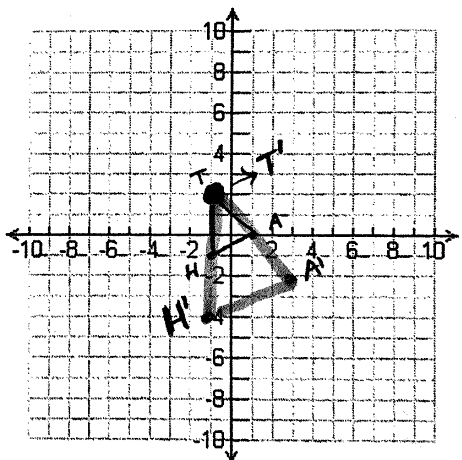
$k<0$?

Dilate by $k = \frac{3}{2}$
Center: $(-4,-2)$



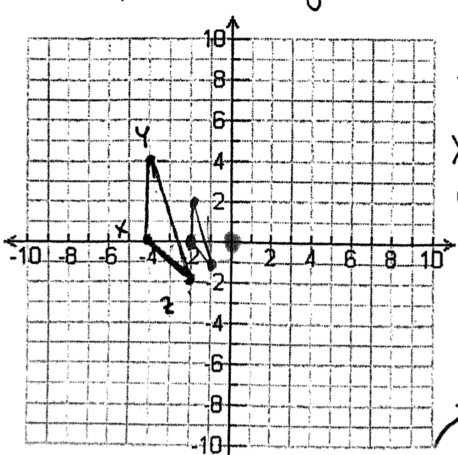
$4 \cdot 1.5 = 6$

Dilate by $k = 2$
Center: $(-1,2)$



* Most dilations are performed from the origin

Dilate by $k = \frac{1}{2}$
from the origin

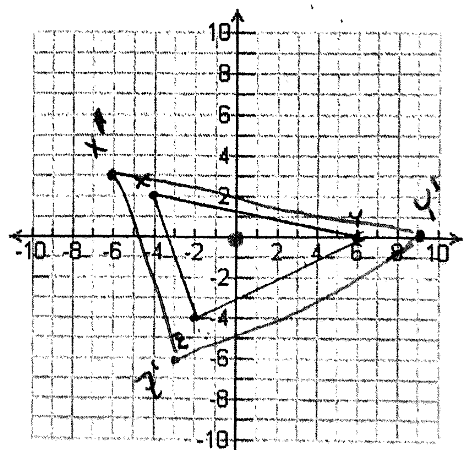


X $(-4,0)$
Y $(-4,4)$
Z $(-2,-2)$
X': $(-2,0)$
Y': $(-2,2)$
Z': $(-1,-1)$

Coordinate Notation:
$(x,y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

→ only works when it's through the origin

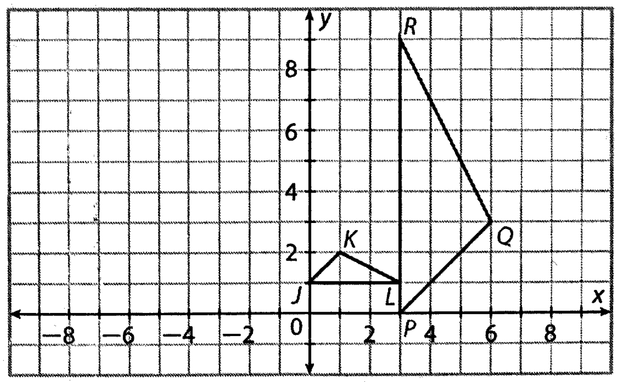
Dilate by $k = 1.5$
from the origin



X': $(-6,3)$
Y': $(9,0)$
Z': $(-3,-6)$

Coordinate Notation:
$(x,y) \rightarrow (1.5x, 1.5y)$

(B) $\triangle JKL$ to $\triangle PQR$



You can map $\triangle JKL$ to $\triangle PQR$ with a reflection across the x-axis followed by a ________ followed by a ☐° counterclockwise rotation about the origin.

Reflection: $(x, y) \rightarrow (x, -y)$ ________: $(x, y) \rightarrow (\quad)$ ☐° counterclockwise rotation: $(x, y) \rightarrow (\quad)$

Reflect

6. Using the figure in Example 3A, describe a single dilation that maps $ABDC$ to $EFHG$.

____

____

7. Using the figure in Example 3B, describe a different sequence of transformations that will map $\triangle JKL$ to $\triangle PQR$.

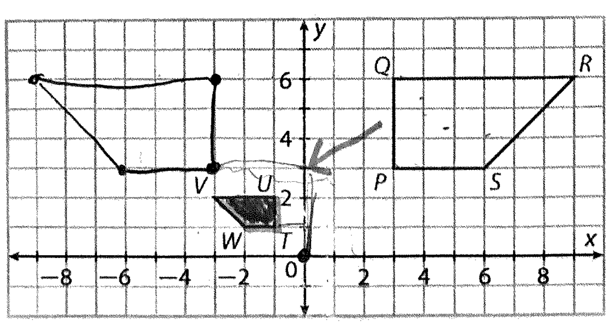
____

____

Your Turn

For each pair of similar figures, find a sequence of similarity transformations that maps one figure to the other. Use coordinate notation to describe the transformations.

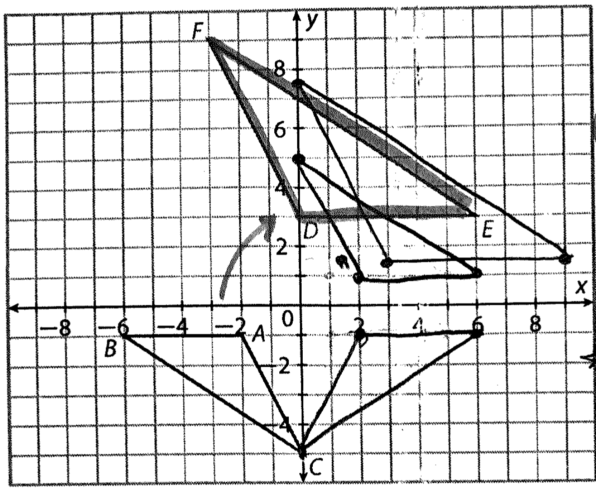
8. $PQRS$ to $TUVW$



① Reflect across y-axis
$(x, y) \rightarrow (-x, y)$

② Dilation scale factor of $\frac{1}{3}$
$(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$

9. $\triangle ABC$ to $\triangle DEF$



① Reflect across x axis
$(x, y) \rightarrow (x, -y)$

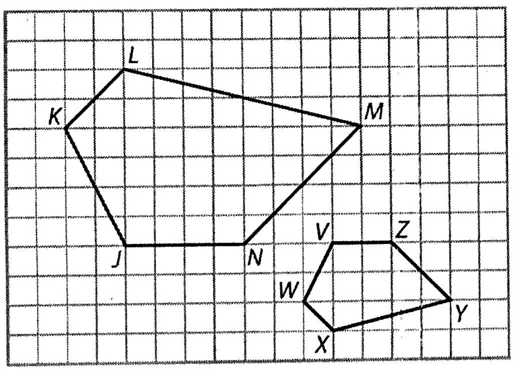
② Reflect across y axis
$(x, y) \rightarrow (-x, y)$

Rotation of 180
$(x, y) \rightarrow (-x, -y)$

③ Dilate scale factor $\frac{3}{2}$
$(x, y) \rightarrow (\frac{3}{2}x, \frac{3}{2}y)$

④ TRANSLATE $(x, y) \rightarrow (x - 3, y + 1.5)$

10. Describe a sequence of similarity transformations that maps *JKLMN* to *VWXYZ*.



Explain 3 Proving All Circles Are Similar

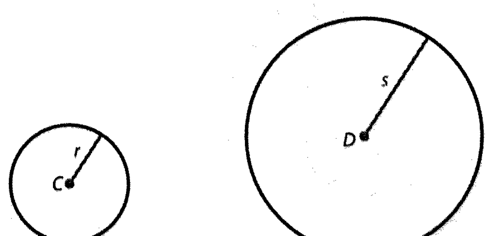
You can use the definition of similarity to prove theorems about figures.

Circle Similarity Theorem

All circles are similar.

Example 3 **Prove the Circle Similarity Theorem.**

Given: Circle C with center C and radius r.
Circle D with center D and radius s.



Prove: Circle C is similar to circle D.

To prove similarity, you must show that there is a sequence of similarity transformations that maps circle C to circle D.