exponent.

Rational Exponents

Objective: Simply expressions with rational/(fraction) exponents

Integers are like $-2, -1, 0, 1, 2$

Rational Numbers are any number that can be written as a fraction

So today we are learning about fraction exponents

$$16^{\frac{1}{2}} \cdot 16^{\frac{1}{2}} = 16^{\frac{1}{2}+\frac{1}{2}} = 16^{1} = 16$$

Therefore $16^{\frac{1}{2}}$ must equal 4.

In fact, $x^{\frac{1}{2}} = \sqrt{x}$

$$64^{\frac{1}{3}} \cdot 64^{\frac{1}{3}} \cdot 64^{\frac{1}{3}} = 64^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 64^{1} = 64$$

So $64^{\frac{1}{3}} = 4$

In fact, $x^{\frac{1}{3}} = \sqrt[3]{x}$

Fraction Exponents with a numerator of 1

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

"A number to the $\frac{1}{n}$ power is equal to the n^{th} root of that number"

Examples:

$$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$$

$$64^{\frac{1}{6}} = \sqrt[6]{64} = 2$$

$$25^{\frac{1}{2}} = \sqrt{25} = 5$$

$$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$

What about $8^{\frac{2}{3}}$?

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

$$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{b}\right)^m$$

<u>OR</u> $b^{\frac{m}{n}} = \left(b^m\right)^{\frac{1}{n}} = \sqrt[n]{b^m}$

<u>Examples:</u>

$16^{\frac{3}{2}}$

$\to \left(16^{\frac{1}{2}}\right)^3 = \left(\sqrt{16}\right)^3 = (4)^3 = 64$

OR

$\to \left(16^3\right)^{\frac{1}{2}} = (4096)^{\frac{1}{2}} = \sqrt{4096} = 64$

$$32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = (2)^3 = 8$$

$$4^{\frac{5}{2}} - 4^{\frac{3}{2}} = \left(\sqrt{4}\right)^5 - \left(\sqrt{4}\right)^3 = 2^5 - 2^3 = 32 - 8 = 24$$

$$25^{\frac{3}{2}} = \left(\sqrt{25}\right)^3 = 5^3 = 125$$

$$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

$(-27)^{\frac{3}{2}}$ vs. $(-27)^{\frac{2}{3}}$

$\left(\sqrt{-27}\right)^3$ vs. $\left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9$

$\downarrow$

Not possible. You cannot square root a negative #

Lesson learned: You can evaluate an odd root for any radicand, but even roots require non-negative radicands. (the # under the radical symbol)

REWRITE each expression using a rational exponent.

$\sqrt[7]{21^5} = (21^5)^{\frac{1}{7}} = 21^{\frac{5}{7}}$

$(\sqrt[7]{21})^5 = (21^{\frac{1}{7}})^5 = 21^{\frac{5}{7}}$

$\sqrt[3]{81^2} = (81^2)^{\frac{1}{3}} = 81^{\frac{2}{3}}$

$(\sqrt[3]{81})^2 = (81^{\frac{1}{3}})^2 = 81^{\frac{2}{3}}$

$\sqrt[4]{45} = 45^{\frac{1}{4}}$

$\sqrt{24} = 24^{\frac{1}{2}}$

$\sqrt{x^3} = (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$

$\sqrt[4]{x^4} = (x^4)^{\frac{1}{4}} = x$

$\sqrt[6]{y^{12}} = (y^{12})^{\frac{1}{6}} = y^2$

Simplify:

$(x^9y^3)^{\frac{1}{3}} = x^{\frac{9}{3}} y^{\frac{3}{3}} = x^3 y$

$\sqrt{a^2b^{10}} = (a^2b^{10})^{\frac{1}{2}} = ab^5$

$\sqrt{9m^6n^4} = (9m^6n^4)^{\frac{1}{2}} = 9^{\frac{1}{2}} m^{\frac{6}{2}} n^{\frac{4}{2}} = 3m^3n^2$

$(25x^4)^{\frac{1}{2}} = 25^{\frac{1}{2}} x^{\frac{4}{2}} = 5x^2$

$\sqrt{x^4y^{12}} = (x^4y^{12})^{\frac{1}{2}} = x^2y^6$

$\sqrt{(9y^2)^2} = [(9y^2)^2]^{\frac{1}{2}} = (9y^2)^{\frac{2}{2}} = 9y^2$

$\sqrt[4]{(xy)^8} = (xy)^{\frac{8}{4}} = (xy)^2 = x^2y^2$