Solving Equations using the Quadratic Formula

You must memorize the Quadratic Formula!

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$\frac{ax^2+bx+c=0}{a}$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$-\frac{c}{a}\cdot\frac{4a}{4a}$$

$$\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{b^2-4ac}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2}$$

Deriving the Quadratic Formula

Many quadratic equations can be solved by graphing, factoring, taking the square root, or completing the square. Some cannot be solved by any of these methods, but you can always use the Quadratic Formula to solve any quadratic equation.

Solve using the Quadratic Formula.

$$6x^2 + 5x - 4 = 0$$

$$\frac{-5 \pm \sqrt{25 - 4(6)(-4)}}{2(6)}$$

$$\frac{-5 \pm \sqrt{121}}{12}$$

$$\frac{-5 \pm 11}{12}$$

$$\frac{-5+11}{12} = \frac{1}{2}$$

$$\frac{-5-11}{12} = -\frac{4}{3}$$

Solve using the Quadratic Formula.

$$x^2 = x + 20$$

$$x^2 - x - 20 = 0$$

$$\frac{-(-1) \pm \sqrt{1 - 4(1)(-20)}}{2(1)}$$

$$\frac{1 \pm \sqrt{81}}{2} \rightarrow \frac{1 \pm 9}{2} \begin{cases} 5 \\ -4 \end{cases}$$

If the quadratic equation is in standard form, the **discriminant** of a quadratic equation is $b^2 - 4ac$, the part of the equation under the radical sign. Recall that quadratic equations can have two, one, or no real solutions. You can determine the number of solutions of a quadratic equation by evaluating its discriminant.

The Discriminant of Quadratic Equation $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$, the equation has two real solutions.

If $b^2 - 4ac = 0$, the equation has one real solution.

If $b^2 - 4ac < 0$, the equation has no real solutions.

Find the number of solutions of each equation using the discriminant.

A.

$3x^2 - 2x + 2 = 0$

$a = 3, b = -2, c = 2$

$b^2 - 4ac$

$(-2)^2 - 4(3)(2)$

$4 - 24$

-20

$b^2 - 4ac$ is negative.

There are no real solutions

B.

$2x^2 + 11x + 12 = 0$

$a = 2, b = 11, c = 12$

$b^2 - 4ac$

$11^2 - 4(2)(12)$

$121 - 96$

25

$b^2 - 4ac$ is positive.

There are two real solutions

C.

$x^2 + 8x + 16 = 0$

$a = 1, b = 8, c = 16$

$b^2 - 4ac$

$8^2 - 4(1)(16)$

$64 - 64$

0

$b^2 - 4ac$ is zero.

There is one real solution

Find the number of solutions of each equation using the discriminant.

a.

$2x^2 - 2x + 3 = 0$

$a = 2, b = -2, c = 3$

$b^2 - 4ac$

$(-2)^2 - 4(2)(3)$

$4 - 24$

-20

$b^2 - 4ac$ is negative.

There are no real solutions

b.

$x^2 + 4x + 4 = 0$

$a = 1, b = 4, c = 4$

$b^2 - 4ac$

$4^2 - 4(1)(4)$

$16 - 16$

0

$b^2 - 4ac$ is zero.

There is one real solution

c.

$x^2 - 9x + 4 = 0$

$a = 1, b = -9, c = 4$

$b^2 - 4ac$

$(-9)^2 - 4(1)(4)$

$81 - 16$

65

$b^2 - 4ac$ is positive.

There are two real solutions